grees of detail, of the power system components which are important to the analysis : synchronous generators and associated controllers, induction motors, non-linear loads of different characteristics and static VAR compensators.

The AESOPS and the implicit inverse iteration algorithms are recently developed techniques and, accordingly, a period of evolution is anticipated. The AESOPS algorithm confines itself to the power system stability problem. The implicit inverse iteration algorithm, on the other hand, will undoubtedly provide an attractive technique for eigensolution in other engineering application areas where very large system matrices are involved.

ACKNOWLEDGEMENT

The author is grateful to Dr. Brian Stott for helpful discussions.

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Discussion

A. J. Calvaer (University of Liege, Belgium) and **M. Stubbe** (Tractionel Belgium): The author is to be congratulated for his paper which presents the state of the art on the field of eigenvalues computation for large power systems. The method that he has implemented (implicit inverse iteration algorithm) occurs efficient).

However, some questions arise as regards practical performances:

1) How to choose the initial eigenvalue estimate? In Table 2 we are looking for for nine electromechanical eigenvalues. The estimates $[0.0 + j0.4 + jk\,0.05, k = 1 \ldots 11]$ converge to seven different eigenvalues. How were determined the initial values $(0.0 + j\,6.7)$ and $0.0 + j\,7.9)$? Is there no risk to multiply the attempts before getting the right estimate?
2) In the ficticious large size system, the eigenvalues are well separate in the complex plane. In a real system, some eigenvalues would be very close to others. In our opinion, this situation could be detrimental to the efficiency of the method, particularly as regards the choice of initial estimates. Has the author some experiment in this field?
3) The "package" of computer programs doesn't contain any unit step response calculation. Can we actually miss it? Has the numerical integration of the linearized system (1) been imagined and has the corresponding computation time been compared with the one of the eigenvalues, eigenvectors and requency response?

Hugh Rudnick (Universidad Católica de Chile, Santiago, Chile): The author is to be congratulated for his valuable contribution to power system steady state stability analysis. The implicit inverse iteration method and the frequency response method have a significant potential for the study of large scale systems.

The discusser is aware of the difficulties of analizing instabilities in large power systems as he performed extensive eigenvalue studies of a real situation occuring in a European system (12) and he suggested the incorporation of system stabilizers which were eventually implemented. The computer programs developed kept storage requirements to a minimum through the use of sparsity techniques and bifactorized matrix inversion methods for the handling of the network equations, and piecewise techniques for the building of the state matrix. Nevertheless, the final state matrix had to be stored and all eigenvalues had to be determined.

The need to determine all eigenvalues can become a serious drawback as the size of the system increases and the modeling complexity grows. The paper is a new contribution in a line of research that has oriented to specific eigenvalue calculations (1, 13, 14). Simplified tools and classical concepts to determine basic unstable modes have also been used to tackle the problem (15, 16). In spite of these developments, many utilities in the world still analize steady state stability with conventional step by step transient stability computer programs, or do not analize it at all, unless operational problems arise.

The author goes into extensive work to adequately model power system load in the system Jacobian matrix. The question is on the practical need to go into that extra complication, considering the usual lack of data on the matter. Moreover, induction motor loads, although contributing with stabilizing torques, have traditionally felt to have a negligible influence in relation to the effect of the excitation loops. Has the author assessed quantitavely the influence of load system stability? Could the frequency response method be useful to show its influence? The same questions apply to the influence of the static var compensators.

A drawback of the proposed algorithm is the need to calculate eigenvectors to associate generator participation and, therefore, evaluate stabilizer locations. How does the author propose to deal with this limitation which is not present in the AESOPS algorithm?

The power frequency response method seems a promising tool and the author is to be commended to pursue further research on it. It would be valuable to explore its applications into the location and tuning of power system stabilizers and the design of complete AVR and governor schemes. The influence on stability of the power transmission through determined system lines could be evaluated with this method, providing a guide in operational procedures. The author's comments would be appreciated.

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F. M. Hughes (UMIST, Manchester, UK.): Dr. Martins is to be congratulated on an excellent paper. The methods described not only enable power system stability to be assessed by efficient eigenvalue calculation, but also enable frequecny responses to be generated between any two selected system variables. This latter facility provides the power engineer with a very powerful tool for the analysis and design of power system controllers. For example, the design of a power system stabilizer for an individual generator can be carried out via standard methods using frequency response mappings which take into account very fully the dynamic characteristics of the rest of the power system. Design can be carried out without having to rely on reduction techniques to reduce the system model to manageable proportions.

Also the ease and speed with which frequency responses can be calculated enables the overall changes in control characteristics with operating condition to be assessed very readily.

It would be interesting to have the author's views on the application of his method to the design of control schemes in the power system context and what future developments he has in mind.

Nelson Martins: The author thanks the discussers for their interest in the paper and the valuable comments and questions. With reference to the points raised by Messrs. Calvaer and Stubbe I offer the following comments:

The number of troublesome electromechanical modes is actually only a small fraction of the very many modes of behavior of the system. Table II of the paper shows the Implicit Inverse Iteration (III) algorithm performance in finding all the electromechanical modes of the New England test system. However, in practice, one is only concerned with the troublesome modes, i.e., those which have low or even negative damping. In our experience, the III algorithm has invariably found the more troublesome modes of large systems.

As suggested by the discussers, we do use an automatic eigenvalue search process in which the estimates are made along the imaginary axis $[0.0 + j(2.0 + 0.1k), k = 1, \ldots, 60]$ when starting a study. Obviously, out of these 60 estimates we may get no more than 10 different eigenvalues (complex conjugate pairs) for a large system. These eigenvalues will always lie closer to the imaginary axis than all the rest.

In subsequent studies, attention will be centered on the troublesome eigenvalues when varying controller settings or operating conditions. The III algorithm proved highly suitable for obtaining root locus plots, by repeatedly calculating the critical eigenvalues as controller parameters are varied by small increments. In these studies, the eigenvalues obtained in a previous run are used as estimates in the next.

Reference 13, listed in Dr. Rudnick's discussion, only came to the author's attention after the submission of this paper. The eigenvalue method presented in Reference 13 is certainly interesting, though little further analysis of convergence conditions is provided. That method obtains in Phase 1 [13] approximations of all system eigenvalues associated with electromechanical oscillations. The author intends to implement that method, using the efficient frequency response algorithm described in this paper, to obtain approximations for the eigenvalues of interest. These approximations will then be used as initial estimates to the III algorithm. The combined use of these two methods are expected to significantly improve program performance.

With reference to the last question of the discussers we inform that new additions are being made to the power system linear analysis package. For the numerical integration of the linearized system equations we have chosen the implicit trapezoidal rule. Higher order numerical integration methods would yield more accurate results at the expense of extra storage and computation time, and were not considered. The numerical integration is performed directly on the power system Jacobian matrix and efficient computation is obtained through use of sparse matrix methods.

With reference to Dr. Rudnick's questions I offer the following comments:

1. The role of loads in the damping of electromechanical oscillations was a topic widely discussed during the CIGRE 1982 Meeting. Paper 31-15 [17] and four other expontaneous contributions [18] made at the general meeting on System Planning unanimously recognized the importance of load modeling for proper simulation of system oscilltions. Mr. Concordia points out in his contribution [18] that the load effects on system damping depend very much on the configuration of the network and the location of the loads in relation to the generators and to the mode shape of the oscillation. Contributions [18] made by Messrs. Y. Laiho, I. A. Erinmez and N. Martins showed that induction motor loading may cause a dramatic increase in oscillation damping, when compared to impedance loads. They also showed that, generally, oscillation damping increases with increasing load inertia.

   It is recognized that there is a strong need for sensitivity analysis to support large scale dynamic stability simulations with information on the parts of the system which are more sensitive to load modeling [19]. Eigenvalue analysis may provide a partial answer to this difficult problem by evaluating the effect of different load representations on the critical oscillation modes of the system.
2. The frequency response method may allow an evaluation of the influence of load characteristics and of static VAR compensators on system electromechanical oscillations. This can be obtained, for example, from plots of synchronizing and damping torques of synchronous machines supplying system loads of different characteristics.
3. When using the AESOPS algorithm, the information on generator participation in a mode shape is also obtained by eigenvector analysis. Eigenvectors are obtained as a by-product in both implicit inverse iteration and AESOPS algorithm.

   The remaining lines of this item are intended to show why the AESOPS algorithm produces eigenvectors as a by-product. The symbols used and equations referred to are those of Section V of the paper.

   At convergence the rotor speed phasors $\omega_i(z^k)$, for all generators, provide information on the mode shape associated with the converged eigenvalue [1]. This fact can be easily visualized by noting that the phasors $\omega_i(z^k)$ are contained in vector $X(z^k)$ and by drawing a comparison with the inverse iteration algorithm. The inverse iteration algorithm, given a good eigenvalue estimate, converges in about two iterations irrespective of the initial guess for the eigenvector.

   The product b. $T_x(z^k)$ of Equation 19 can be seen as the initial guess for an eigenvector in the inverse iteration algorithm (see Equation 5.a). Assuing that z^k is a very good eigenvalue estimate, as happens near convergence, the calculated vector $X(z^k)$, equivalent to the result of one iteration of the inverse iteration algorithm, is equal to the correct eigenvector for practical purposes.

Concerning the points raised by Messrs. Hughes and Rudnick on the proposed frequency response algorithm, I wish to make the following comments:

The discussers already pointed out some of the more important uses for the frequency response algorithm. The author lists below the main applications he found for this algorithm.

1. Design of power system controllers by frequency response mappings, considering the dynamic effects of the whole multimachine system. These controllers can be associated with a generating plant, static VAR compensator or a HVDC link.
2. Evaluation of the dynamic interaction: a) between control loops in a generating plant or in a HVDC link; b) between adjacent generators or static VAR compensators. These studies would be based on multivariable frequency response mappings [20] obtained by the proposed algorithm, and would allow design of multivariable controllers when necessary.
3. Computer duplication of field measurements for data validation or parameter estimation.
4. Improved methods for eigenvalue calculation, as mentioned in the author's reply to Messrs. Calvaer and Stubbe.

I close by thanking again the discussers for the considerable attention they have devoted to this paper.

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